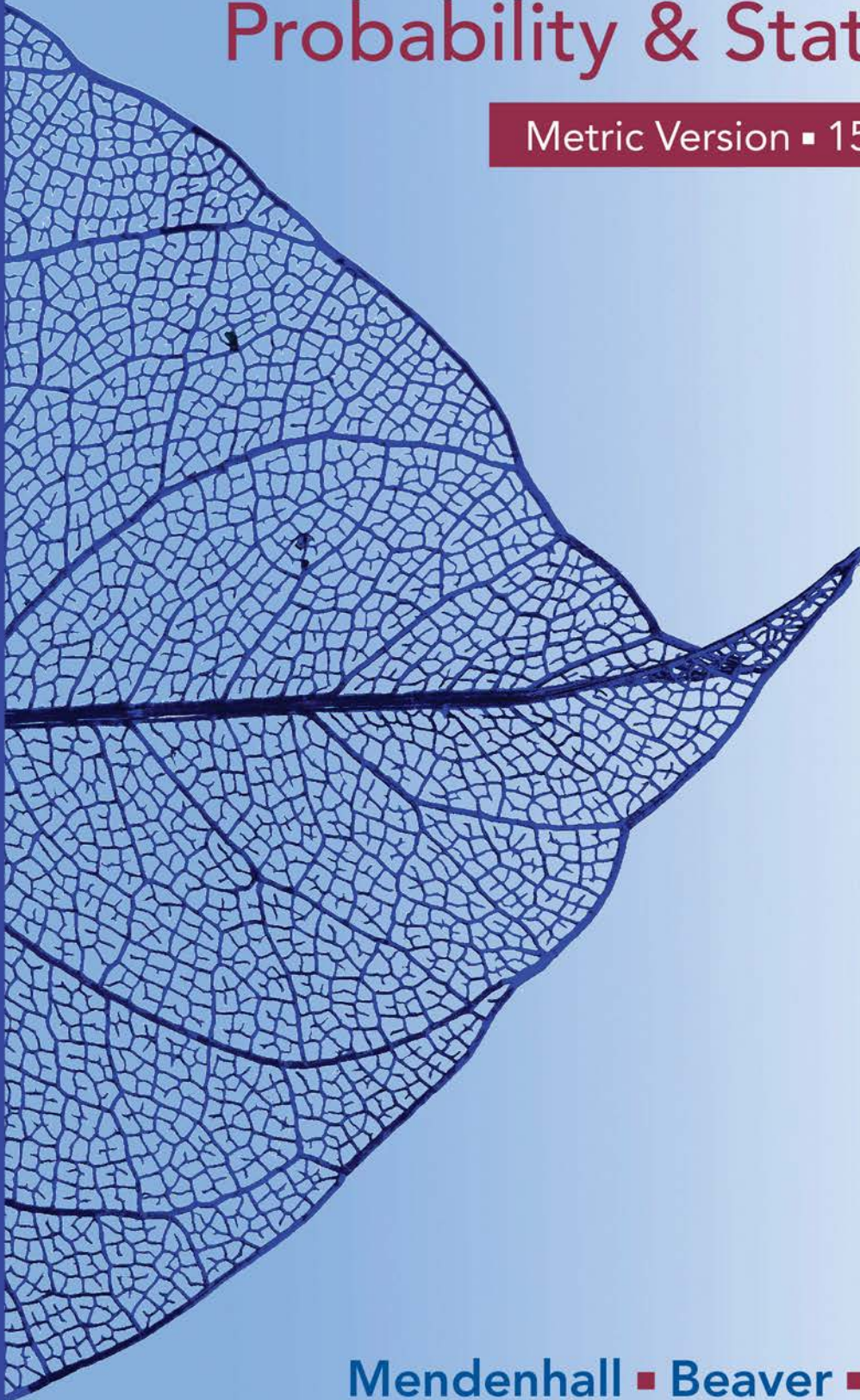


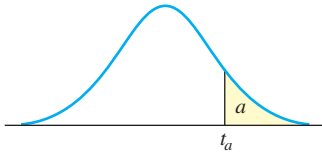
# Introduction to Probability & Statistics

Metric Version ■ 15th Edition

METRIC VERSION



Mendenhall ■ Beaver ■ Beaver



■ Table 4 Critical Values of  $t$ , pages 692–963

$df$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$df$
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.309	1.696	2.040	2.453	2.744	31
32	1.309	1.694	2.037	2.449	2.738	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.434	2.719	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.704	40
45	1.301	1.679	2.014	2.412	2.690	45
50	1.299	1.676	2.009	2.403	2.678	50
55	1.297	1.673	2.004	2.396	2.668	55
60	1.296	1.671	2.000	2.390	2.660	60
65	1.295	1.669	1.997	2.385	2.654	65
70	1.294	1.667	1.994	2.381	2.648	70
80	1.292	1.664	1.990	2.374	2.639	80
90	1.291	1.662	1.987	2.368	2.632	90
100	1.290	1.660	1.984	2.364	2.626	100
200	1.286	1.653	1.972	2.345	2.601	200
300	1.284	1.650	1.968	2.339	2.592	300
400	1.284	1.649	1.966	2.336	2.588	400
500	1.283	1.648	1.965	2.334	2.586	500
inf.	1.282	1.645	1.96	2.326	2.576	inf.

Source: Percentage points calculated using *Minitab* software.

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EDITION  
**15**

# Introduction to Probability and Statistics

## Metric Version

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1925–2009

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Fifteenth Edition, Metric Version**

**William Mendenhall, III, Robert J.  
Beaver, Barbara M. Beaver**

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# Preface

Every time you pick up a newspaper or a magazine, watch TV, or scroll through Facebook, you encounter statistics. Every time you fill out a questionnaire, register at an online website, or pass your grocery rewards card through an electronic scanner, your personal information becomes part of a database containing your personal statistical information. You can't avoid it! In this digital age, data collection and analysis are part of our day-to-day activities. If you want to be an educated consumer and citizen, you need to understand how statistics are used and misused in our daily lives.

This international metric version is designed for classrooms and students outside of the United States. The units of measurement used in selected examples and exercises have been changed from U.S. Customary units to metric units. We did not update problems that are specific to U.S. Customary units, such as passing yards in football or data related to specific publications.

## The Secret to Our Success

The first college course in introductory statistics that we ever took used *Introduction to Probability and Statistics* by William Mendenhall. Since that time, this text—currently in the fifteenth edition—has helped generations of students understand what statistics is all about and how it can be used as a tool in their particular area of application. The secret to the success of *Introduction to Probability and Statistics* is its ability to blend the old with the new. With each revision we try to build on the strong points of previous editions, and to look for new ways to motivate, encourage, and interest students using new technologies.

## Hallmark Features of the Fifteenth Edition

The fifteenth edition keeps the traditional outline for the coverage of descriptive and inferential statistics used in previous editions. This revision maintains the straightforward presentation of the fourteenth edition. We have continued to simplify the language in order to make the text more readable—without sacrificing the statistical integrity of the presentation. We want students to understand how to apply statistical procedures, and also to understand

- how to meaningfully describe real sets of data
- how to explain the results of statistical tests in a practical way
- how to tell whether the assumptions behind statistical tests are valid
- what to do when these assumptions have been violated

## Exercises

As with all previous editions, the variety and number of real applications in the exercise sets is a major strength of this edition. We have revised the exercise sets to provide new and

interesting real-world situations and real data sets, many of which are drawn from current periodicals and journals. The fifteenth edition contains over 1900 exercises, many of which are new to this edition. Exercises are graduated in level of difficulty; some, involving only basic techniques, can be solved by almost all students, while others, involving practical applications and interpretation of results, will challenge students to use more sophisticated statistical reasoning and understanding. Exercises have been rearranged to provide a more even distribution of exercises within each chapter and a new numbering system has been introduced, so that numbering begins again with each new section.


## Organization and Coverage

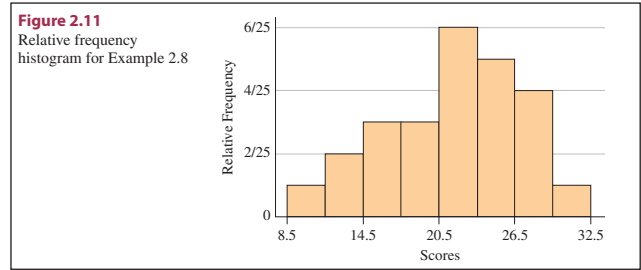
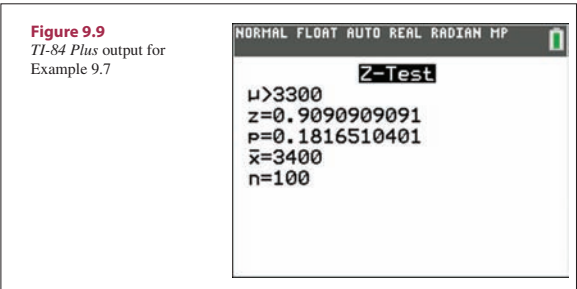
We believe that Chapters 1 through 10—with the possible exception of Chapter 3—should be covered in the order presented. The remaining chapters can be covered in any order. The analysis of variance chapter precedes the regression chapter, so that the instructor can present the analysis of variance as part of a regression analysis. Thus, the most effective presentation would order these three chapters as well.

Chapters 1–3 present descriptive data analysis for both one and two variables, using *MINITAB 18*, *Microsoft Excel 2016*<sup>®</sup>, and *TI-83/84 Plus* graphics. Chapter 4 includes a full presentation of probability. The last section of Chapter 4 in the fourteenth edition of the text, “Discrete Random Variables and Their Probability Distributions” has been moved to become the first section in Chapter 5. As in the fourteenth edition, the chapters on analysis of variance and linear regression include both calculational formulas and computer printouts in the basic text presentation. These chapters can be used with equal ease by instructors who wish to use the “hands-on” computational approach to linear regression and ANOVA and by those who choose to focus on the interpretation of computer-generated statistical printouts. This edition includes expanded coverage of the uniform and exponential distributions in Chapter 5 and normal probability plots for assessing normality in Chapter 7, in addition to an expanded *t*-table (Table 4 in Appendix I). New topics in Chapter 13 include best subsets regression procedures and binary logistic regression.

One important feature in the hypothesis testing chapters involves the emphasis on *p*-values and their use in judging statistical significance. With the advent of computer-generated *p*-values, these probabilities have become essential in reporting the results of a statistical analysis. As such, the observed value of the test statistic and its *p*-value are presented together at the outset of our discussion of statistical hypothesis testing as equivalent tools for decision-making. Statistical significance is defined in terms of preassigned values of  $\alpha$ , and the *p*-value approach is presented as an alternative to the *critical value approach* for testing a statistical hypothesis. Examples are presented using both the *p*-value and *critical value* approaches to hypothesis testing. Discussion of the practical interpretation of statistical results, along with the difference between statistical significance and practical significance, is emphasized in the practical examples in the text.

## Special Features of the Fifteenth Edition

- **NEED TO KNOW. . .**: This edition again includes highlighted sections called “NEED TO KNOW. . .” and identified by this icon.  **Need to Know...** These sections provide information consisting of definitions, procedures, or step-by-step hints on problem solving for specific questions such as “NEED TO KNOW... How to Construct a Relative Frequency Histogram?” or “NEED TO KNOW... How to Decide Which Test to Use?”
- Graphical and numerical data description includes both traditional and EDA methods, using computer graphics generated by *MINITAB 18* for Windows and *MS Excel 2016*.
- Calculator screen captures from the *TI-84 Plus* calculator have been used for several examples, allowing students to access this option for data analysis.



	E	F	G	H
	Front Leg Room		Rear Leg Room	
Mean		41.9	Mean	28.350
Standard Error		0.221	Standard Error	0.409
Median		41.750	Median	28
Mode		41.500	Mode	28
Standard Deviation		0.699	Standard Deviation	1.292
Sample Variance		0.489	Sample Variance	1.669
Kurtosis		2.456	Kurtosis	-0.163
Skewness		1.353	Skewness	-0.021
Range		2.5	Range	4
Minimum		41	Minimum	26
Maximum		43.5	Maximum	30
Sum		419	Sum	283.5
Count		10	Count	10

- All examples and exercises in the text that contain printouts or calculator screen captures are based on *MINITAB 18*, *MS Excel 2016*, or the *TI-84 Plus* calculator. These outputs are provided for some exercises, while other exercises require the student to obtain solutions without using a computer.

Name	Length (km)	Name	Length (km)
Superior	560	Titicaca	195
Victoria	334	Nicaragua	163
Huron	330	Athabasca	333
Michigan	491	Reindeer	229
Aral Sea	416	Tonle Sap	112
Tanganyika	672	Turkana	246
Baykal	632	Issyk Kul	184
Great Bear	307	Torrens	208
Nyasa	576	Vänern	146
Great Slave	477	Nettilling	107
Erie	386	Winnipegosis	226
Winnipeg	426	Albert	160
Ontario	309	Nipigon	115
Balkhash	602	Gairdner	144
Ladoga	198	Urmia	144
Maracaibo	213	Manitoba	224
Onega	232	Chad	280
Eyre	144		

Source: *The World Almanac and Book of Facts 2017*

- Use a stem and leaf plot to describe the lengths of the world's major lakes.
- Use a histogram to display these same data. How does this compare to the stem and leaf plot in part a?
- Are these data symmetric or skewed? If skewed, what is the direction of the skewing?

**DATA SET 6. Gulf Oil Spill Cleanup** On April 20, 2010, the United States experienced a major environmental disaster when a Deepwater Horizon drilling rig exploded in the Gulf of Mexico. The number of personnel and equipment used in the Gulf oil spill cleanup, beginning May 2, 2010 (Day 13) through June 9, 2010 (Day 51) is given in the following table.<sup>17</sup>

- Use a bar graph to show the percentage of federal Gulf fishing areas closed.
- Use a line chart to show the amounts of dispersants used. Is there any underlying straight line relationship over time?

**DATA SET 7. Election Results** The 2016 election was a race in which Donald Trump defeated Hillary Clinton and other candidates, winning 304 electoral votes, or 57% of the 538 available. However, Trump only won 46.1% of the popular vote, while Clinton won 48.2%. The popular vote (in thousands) for Donald Trump in each of the 50 states is listed as follows<sup>18</sup>:

AL	1319	HI	129	MA	1091	NM	320	SD	228
AK	163	ID	409	MI	2280	NY	2820	TN	1523
AZ	1252	IL	2146	MN	1323	NC	2363	TX	4685
AR	685	IN	1557	MS	701	ND	217	UT	515
CA	4484	IA	801	MO	1595	OH	2841	VT	95
CO	1202	KS	671	MT	279	OK	949	VA	1769
CT	673	KY	1203	NE	496	OR	782	WA	1222
DE	185	LA	1179	NV	512	PA	2971	WV	489
FL	4618	ME	336	NH	346	RI	181	WI	1405
GA	2089	MD	943	NJ	1602	SC	1155	WY	174

- By just looking at the table, what shape do you think the distribution for the popular vote by state will have?
- Draw a relative frequency histogram to describe the distribution of the popular vote for President Trump in the 50 states.
- Did the histogram in part b confirm your guess in part a? Are there any outliers? How can you explain them?

**TECHNOLOGY TODAY**

## The Role of Computers and Calculators in the Fifteenth Edition—Technology Today

Computers and scientific or graphing calculators are now common tools for college students in all disciplines. Most students are accomplished users of word processors, spreadsheets, and databases, and they have no trouble navigating through software packages in the Windows environment. Many own either a scientific or a graphing calculator, very often one of the many calculators made by Texas Instruments.<sup>TM</sup> We believe, however, that advances in computer technology should not turn statistical analyses into a “black box.” Rather, we choose to use the computational shortcuts that modern technology provides to give us more time to emphasize statistical reasoning as well as the understanding and interpretation of statistical results.

In this edition, students will be able to use computers both for standard statistical analyses and as a tool for reinforcing and visualizing statistical concepts. Both *MS Excel 2016* and *MINITAB 18* are used exclusively as the computer packages for statistical analysis along with procedures available using the *TI-83* or *TI-84 Plus* calculators. However, we have chosen to isolate the instructions for generating computer and calculator output into individual sections called *Technology Today* at the end of each chapter. Each discussion uses numerical examples to guide the student through the *MS Excel* commands and option necessary for the procedures presented in that chapter, and then present the equivalent steps and commands needed to produce the same or similar results using *MINITAB* and the *TI-83/84 Plus*. We have included screen captures from *MS Excel*, *MINITAB 18*, and the *TI-84 Plus*, so that the student can actually work through these sections as “mini-labs.”

If you do not need “hands-on” knowledge of *MINITAB*, *MS Excel*, or the *TI-83/84 Plus*, or if you are using another calculator or software package, you may choose to skip these sections and simply use the printouts as guides for the basic understanding of computer or calculator outputs.

**TECHNOLOGY TODAY**

**Numerical Descriptive Measures in Excel**

*MS Excel* provides most of the basic descriptive statistics presented in Chapter 2 using a single command on the **Data** tab. Other descriptive statistics can be calculated using the **Function Library** group on the **Formulas** tab.

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**EXAMPLE 2.17** The following data are the front and rear leg rooms (in inches) for 10 different compact sports utility vehicles<sup>13</sup>:

Make & Model	Front Leg Room	Rear Leg Room
Chevrolet Equinox	42.5	30.0
Ford Escape	41.5	28.0
Hyundai Tucson	41.5	28.0
Jeep Cherokee	43.5	30.0
Jeep Compass	41.5	28.0

**Numerical Descriptive Measures in MINITAB**

*MINITAB* provides most of the basic descriptive statistics presented in Chapter 2 using a single command in the drop-down menus.

---

**EXAMPLE 2.18** The following data are the front and rear leg rooms (in inches) for 10 different compact sports utility vehicles<sup>13</sup>:

Make & Model	Front Leg Room	Rear Leg Room
Chevrolet Equinox	42.5	30.0
Ford Escape	41.5	28.0
Hyundai Tucson	41.5	28.0
Jeep Cherokee	43.5	30.0

**Numerical Descriptive Measures on the TI-83/84 Plus Calculators**

The *TI-83/84 Plus* calculators can be used to calculate descriptive statistics and create box plots using the **stat** ► **CALC** and **2nd** ► **stat plot** commands.

---

**EXAMPLE 2.19** The following data are the front and rear leg rooms (in inches) for 10 different compact sports utility vehicles<sup>13</sup>:

Make & Model	Front Leg Room	Rear Leg Room
Chevrolet Equinox	42.5	30.0
Ford Escape	41.5	28.0

## Study Aids

The many and varied exercises in the text provide the best learning tool for students embarking on a first course in statistics. The answers to all odd-numbered exercises are given in the back of the text. Each application exercise has a title, making it easier for students and instructors to immediately identify both the context of the problem and the area of application. All of the basic exercises have been rewritten and all of the applied exercises restructured according to increasing difficulty. New exercises have been introduced, dated exercises have been deleted, and a new numbering system has been introduced within each section.

### The Basics

**Normal Approximation?** Can the normal approximation be used to approximate probabilities for the binomial random variable  $x$ , with values for  $n$  and  $p$  given in Exercises 1–4? If not, is there another approximation that you could use?

1.  $n = 25$  and  $p = .6$
2.  $n = 45$  and  $p = .05$
3.  $n = 25$  and  $p = .3$
4.  $n = 15$  and  $p = .5$

**Using the Normal Approximation** Find the mean and standard deviation for the binomial random variable  $x$

12.  $P(x \geq 6)$  and  $P(x > 6)$  when  $n = 15$  and  $p = .5$
13.  $P(4 \leq x \leq 6)$  when  $n = 25$  and  $p = .2$
14.  $P(x \geq 7)$  and  $P(x = 5)$  when  $n = 20$  and  $p = .3$
15.  $P(x \geq 10)$  when  $n = 20$  and  $p = .4$

### Applying the Basics

16. A *USA Today* snapshot found that 47% of Americans associate “recycling” with Earth Day.<sup>9</sup> Suppose a random sample of  $n = 50$  adults are polled and that the

Students should be encouraged to use the “**NEED TO KNOW. . .**” sections as they occur in the text. The placement of these sections is intended to answer questions as they would normally arise in discussions. In addition, there are numerous hints called “**NEED A TIP?**” that appear in the margins of the text. The tips are short and concise.

#### ● Need a Tip?

Parameter  $\leftrightarrow$  Population  
Statistic  $\leftrightarrow$  Sample

In the previous three chapters, you have learned a lot about probability distributions, such as the binomial and normal distributions. The shape of the normal distribution is determined by its mean  $\mu$  and its standard deviation  $\sigma$ , while the shape of the binomial distribution is determined by  $p$ . These numerical descriptive measures—called **parameters**—are needed to calculate the probability of observing sample results.

In practical situations, you may be able to decide which *type* of probability distribution to use as a model, but the values of the *parameters* that specify its *exact form* are unknown. Here are two examples:

Finally, sections called **Key Concepts and Formulas** appear in each chapter as a review in outline form of the material covered in that chapter.

## CHAPTER REVIEW

### Key Concepts and Formulas

#### I. Measures of the Center of a Data Distribution

1. Arithmetic mean (mean) or average
  - a. Population:  $\mu$
  - b. Sample of  $n$  measurements:  $\bar{x} = \frac{\sum x_i}{n}$
2. Median; **position** of the median =  $.5(n + 1)$
3. Mode
4. The median may be preferred to the mean if the data are highly skewed.

2. The Empirical Rule can be used only for relatively mound-shaped data sets. Approximately 68%, 95%, and 99.7% of the measurements are within one, two, and three standard deviations of the mean, respectively.

#### IV. Measures of Relative Standing

1. Sample z-score:  $z = \frac{x - \bar{x}}{s}$
2.  $p$ th percentile;  $p\%$  of the measurements are smaller, and  $(100 - p)\%$  are larger.
3. Lower quartile,  $Q_1$ ; **position** of  $Q_1 = .25(n + 1)$

## Instructor Resources

### WebAssign



WebAssign for Mendenhall/Beaver/Beaver's *Introduction to Probability and Statistics*, 15th Edition, Metric Version is a flexible and fully customizable online instructional solution that puts powerful tools in the hands of instructors, empowering you to deploy assignments, instantly assess individual student and class performance, and help your students master the course concepts. With WebAssign's powerful digital platform and *Introduction to Probability and Statistics's* specific content, you can tailor your course with a wide range of assignment settings, add your own questions and content, and access student and course analytics and communication tools.

### MindTap Reader

Available via WebAssign, MindTap Reader is Cengage's next-generation eBook. MindTap Reader provides robust opportunities for students to annotate, take notes, navigate, and interact with the text. Instructors can edit the text and assets in the Reader, as well as add videos or URLs.

### Cognero

Cengage Learning Testing, powered by Cognero, is a flexible, online system that allows you to import, edit, and manipulate content from the text's Test Bank or elsewhere—including your own favorite test questions; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want.

### Instructor Solutions Manual

This time-saving online manual provides complete solutions to all the problems in the text. You can download the solutions manual from the Instructor Companion Website.

### Instructor Companion Website

Everything you need for your course in one place! This collection of book-specific class tools is available online via [www.cengage.com/login](http://www.cengage.com/login). Access and download PowerPoint presentations, images, Instructor Solutions Manual, data sets, and more.

### SnapStat

Tell the story behind the numbers with SnapStat in WebAssign. Designed with students to bring stats to life, SnapStat uses interactive visuals to perform complex analysis online. Labs and Projects in WebAssign allow students to crunch their own data or choose from pre-existing data sets to get hands-on with technology and see for themselves that Statistics is much more than just numbers.

## Student Resources

### WebAssign



WebAssign for Mendenhall/Beaver/Beaver's *Introduction to Probability and Statistics*, 15th Edition, Metric Version lets you prepare for class with confidence. Its online learning



platform for your math, statistics, and science courses helps you practice and absorb what you learn. Videos and tutorials walk you through concepts when you're stuck, and instant feedback and grading let you know where you stand—so you can focus your study time and perform better on in-class assignments. Study smarter with WebAssign!

## MindTap Reader

Available via WebAssign, MindTap Reader is Cengage's next-generation eBook. MindTap Reader provides robust opportunities for students to annotate, take notes, navigate, and interact with the text. Annotations captured in MindTap are automatically tied to the Notepad app, where they can be viewed chronologically and in a cogent, linear fashion.

## Online Technology Guides

Online Technology Guides, accessed via [www.cengage.com](http://www.cengage.com), provide step-by-step instructions for completing problems using common statistical software.

## SnapStat

Learn the story behind the numbers with SnapStat in WebAssign. Designed with students to bring stats to life, SnapStat uses interactive visuals to perform complex analysis online. Labs and Projects in WebAssign allow you to crunch your own data or choose from pre-existing data sets to get hands-on with technology and see for yourself that Statistics is much more than just numbers.

## Acknowledgments

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*Robert J. Beaver*  
*Barbara M. Beaver*



# Introduction

## What Is Statistics?



Andrea Ricordi, Italy/Moment/Getty Images

What is statistics? Have you ever met a statistician? Do you know what a statistician does? Maybe you are thinking of the person who sits in the broadcast booth at the Rose Bowl, recording the number of pass completions, yards rushing, or interceptions thrown on New Year's Day. Or maybe just hearing the word *statistics* sends a shiver of fear through you. You might think you know nothing about statistics, but almost every time you turn on the news or scroll through your favorite news app, you will find statistics in one form or other! Here are some examples that we found just before the 2017 November elections:

- **Northam Heads Into Virginia Governor's Race With A Small Lead.** The first major statewide elections since President Trump was inaugurated take place on Tuesday...And while the race's final result by itself isn't likely to tell us much about the national political environment, it *is likely* to have a big effect on the 2018 midterms. Polls show a fairly close race, with Northam slightly favored to win [over Ed Gillespie]. An average of the last 10 surveys give Northam a 46 percent-to-43 percent advantage. Over the past month, there has been a tightening of the race, with Gillespie closing what had been a 6-point lead. In the individual polls, though, there is a fairly wide spread. Northam has led by as much as 17 percentage points (a Quinnipiac University survey) and has trailed by as much as 8 points (a Hampton University poll).<sup>1</sup>

—www.fivethirtyeight.com

- **Why Trump Has a Lock on the 2020 GOP Nomination.** In interviews with nearly three-dozen GOP strategists and fundraisers over the past several tumultuous weeks, virtually everyone told me that...they expect Trump to coast to the GOP nomination in 2020...the hurdles to a 2020 primary challenge are vivid when considering a recent *Washington Post/ABC News* poll that found 91% of Trump voters said they'd vote for him again...This *ABC News/Washington Post* poll was conducted by landline and cellular telephone Oct. 29-Nov. 1, 2017, in English and Spanish, among a random national sample of 1005 adults. Results have a margin of sampling error of 3.5 points, including the design effect.<sup>2</sup>

—www.cnn.com

Articles similar to these can be found in all forms of news media, and, just before a presidential or congressional election, a new poll is reported almost every day. These articles are very familiar to us; however, they might leave you with some unanswered questions. How were the people in the poll selected? Will these people give the same response tomorrow? Will they give the same response on election day? Will they even vote? Are these people representative of all those who will vote on election day? It is the job of a statistician to ask these questions and to find answers for them in the language of the poll.

#### Most Believe “Cover-Up” of JFK Assassination Facts

A majority of the public believes the assassination of President John F. Kennedy was part of a larger conspiracy, not the act of one individual. In addition, most Americans think there was a cover-up of facts about the 1963 shooting. Almost 50 years after JFK’s assassination, a FOX news poll shows many Americans disagree with the government’s conclusions about the killing. The **Warren Commission** found that **Lee Harvey Oswald** acted alone when he shot Kennedy, but 66 percent of the public today think the assassination was “part of a larger conspiracy” while only 25 percent think it was the “act of one individual.”

“For older Americans, the Kennedy assassination was a traumatic experience that began a loss of confidence in government,” commented Opinion Dynamics President John Gorman. “Younger people have grown up with movies and documentaries that have pretty much pushed the ‘conspiracy’ line. Therefore, it isn’t surprising there is a fairly solid national consensus that we still don’t know the truth.”

(The poll asked): “Do you think that we know all the facts about the assassination of President John F. Kennedy or do you think there was a cover-up?”

	We Know All the Facts (%)	There Was a Cover-Up	(Not Sure)
All	14	74	12
Democrats	11	81	8
Republicans	18	69	13
Independents	12	71	17

—www.foxnews.com<sup>3</sup>

When you see an article like this one, do you simply read the title and the first paragraph, or do you read further and try to understand the meaning of the numbers? How did the authors get these numbers? Did they really interview every American with each political affiliation? It is the job of the statistician to answer some of these questions.

#### Hot News: 98.6°F Not Normal

After believing for more than a century that 98.6°F was the normal body temperature for humans, researchers now say normal is not normal anymore.

For some people at some hours of the day, 99.9°F could be fine. And readings as low as 96°F turn out to be highly human.

The 98.6°F standard was derived by a German doctor in 1868. Some physicians have always been suspicious of the good doctor’s research. His claim: 1 million readings—in an epoch without computers.

So Mackowiak & Co. took temperature readings from 148 healthy people over a three-day period and found that the mean temperature was 98.2°F. Only 8 percent of the readings were 98.6°F.

—*The Press-Enterprise*<sup>4</sup>

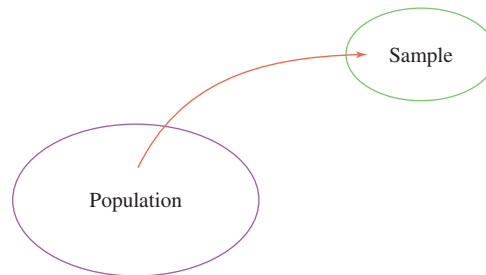
What questions do you have when you read this article? How did the researcher select the 148 people, and how can we be sure that the results based on these 148 people are accurate when applied to the general population? How did the researcher arrive at the normal “high” and “low” temperatures given in the article? How did the German doctor record 1 million temperatures in 1868? This is another statistical problem with an application to everyday life.

Statistics is a branch of mathematics that has applications in almost every part of our daily life. It is a new and unfamiliar language for most people, however, and, like any

new language, statistics can seem overwhelming at first glance. But once the language of statistics is learned and understood, it provides a powerful tool for data analysis in many different fields of application.

## The Population and the Sample

In the language of statistics, one of the most basic concepts is **sampling**. In most statistical problems, a specified number of measurements or data—a **sample**—is drawn from a much larger body of measurements, called the **population**.



For the body-temperature experiment, the sample is the set of body-temperature measurements for the 148 healthy people chosen by the experimenter. We hope that the sample is representative of a much larger body of measurements—the population—the body temperatures of all healthy people in the world!

Which is more important to us, the sample or the population? In most cases, we are interested primarily in the population, but identifying each member of the population may be difficult or impossible. Imagine trying to record the body temperature of every healthy person on earth or the presidential preference of every registered voter in the United States! Instead, **we try to describe or predict the behavior of the population on the basis of information obtained from a representative sample from that population.**

The words *sample* and *population* have two meanings for most people. For example, you read that a Gallup poll conducted in the United States was based on a sample of 1823 people. Presumably, each person interviewed is asked a particular question, and that person's response represents a single measurement in the sample. Is the sample the set of 1823 people, or is it the 1823 responses that they give?

In statistics, we distinguish between the set of objects on which the measurements are taken and the measurements themselves. To experimenters, the objects on which measurements are taken are called **experimental units**. The sample survey statistician calls them **elements of the sample**.

## Descriptive and Inferential Statistics

When first presented with a set of measurements—whether a sample or a population—you need to find a way to organize and summarize it. The branch of statistics that gives us tools for describing sets of measurements is called **descriptive statistics**. You have seen descriptive statistics in many forms: bar charts, pie charts, and line charts presented by a political candidate; numerical tables in the media; or the average rainfall amounts on your favorite weather app. Computer-generated graphics and numerical summaries are commonplace in our everyday communication.

**DEFINITION**

**Descriptive statistics** are procedures used to summarize and describe the important characteristics of a set of measurements.

If the set of measurements is the entire population, you need only to draw conclusions based on the descriptive statistics. However, it might be too expensive or too time consuming to identify each member of the population. Maybe listing the entire population would destroy it—for example, measuring the amount of force required to cause a football helmet crack. For these or other reasons, you may have only a sample from the population. By looking at the sample, you want to answer questions about the population as a whole. The branch of statistics that deals with this problem is called **inferential statistics**.

**DEFINITION**

**Inferential statistics** are procedures used to make inferences (that is, draw conclusions, make predictions, make decisions) about a population from information contained in a sample drawn from this population.

The **objective of inferential statistics** is to make inferences about a population from information contained in a sample.

## Achieving the Objective of Inferential Statistics: The Necessary Steps

How can you make inferences about a population using information contained in a sample? The task becomes simpler if you organize the problem into a series of logical steps.

- 1. Specify the questions to be answered and identify the population of interest.** In the Virginia election poll, the objective is to determine who will get the most votes on election day. So, the population of interest is the set of all votes in the Virginia election. When you select a sample, it is important that the sample be representative of *this* population, not the population of voter preferences on some day prior to the election.
- 2. Decide how to select the sample.** This is called the *design of the experiment* or the *sampling procedure*. Is the sample representative of the population of interest? For example, if a sample of registered voters is selected from the city of San Francisco, will this sample be representative of all voters in California? Will it be the same as a sample of “likely voters”—those who are likely to actually vote in the election? Is the sample large enough to answer the questions posed in step 1 without wasting time and money on additional information? A good sampling design will answer the questions posed with minimal cost to the experimenter.
- 3. Select the sample and analyze the sample information.** No matter how much information the sample contains, you must use an appropriate method of analysis to obtain it. Many of these methods, which depend on the sampling procedure in step 2, are explained in the text.

4. **Use the information from step 3 to make an inference about the population.** Many different procedures can be used to make this inference, and some are better than others. For example, 10 different methods might be available to estimate human response to an experimental drug, but one procedure might be more accurate than others. You should use the best inference-making procedure available (many of these are explained in the text).
5. **Determine the reliability of the inference.** Since you are using only a fraction of the population in drawing the conclusions described in step 4, you might be wrong! If an agency conducts a statistical survey for you and estimates that your company's product will gain 34% of the market this year, how much confidence can you place in this estimate? Is this estimate accurate to within 1, 5, or 20 percentage points? Is it reliable enough to be used in setting production goals? Every statistical inference should include a measure of reliability that tells you how much confidence you have in the inference.

Now that you have learned a few basic terms and concepts, we again pose the question asked at the beginning of this discussion: Do you know what a statistician does? The statistician's job is to carry out all of the preceding steps.

## Keys for Successful Learning

As you begin to study statistics, you will find that there are many new terms and concepts to be mastered. Since statistics is an applied branch of mathematics, many of these basic concepts are mathematical—developed and based on results from calculus or higher mathematics. However, you do not have to be able to prove the results in order to apply them in a logical way. In this text, we use numerical examples and commonsense arguments to explain statistical concepts, rather than more complicated mathematical arguments.

Computers and calculators are now readily available to many students and provide them with an invaluable tool. In the study of statistics, even the beginning student can use packaged programs to perform statistical analyses with a high degree of speed and accuracy. Some of the more common statistical packages available at computer facilities are *MINITAB*<sup>TM</sup>, *SAS*, and *SPSS*. Personal computers and laptops will support *MINITAB*, *MS EXCEL*, *JMP*, and others. Many students are familiar with the *TI-83* or *TI-84 Plus* calculators, that have many built-in statistics functions. There are even online statistical programs and interactive “applets” that students can use.

These programs, called **statistical software**, differ in the types of analyses available, the options within the programs, and the forms of printed results (called **output**). However, they are all similar. In this book, we use both *MINITAB* and *Microsoft Excel* as statistical tools. Understanding the basic output of these packages will help you interpret the output from other software systems. Similarly, understanding the results shown on your *TI-83* or *TI-84 Plus* calculator will make understanding a different calculator much easier.

At the end of most chapters, you will find a section called “*Technology Today*.” These sections present numerical examples to guide you through the *MINITAB*, *MS Excel*, and *TI-83/84 Plus* commands and options that are used for the procedures in that chapter. If you are using *MINITAB*, *MS Excel*, or your *TI-83/84 Plus* calculator in a lab or home setting, you may want to work through this section using your own computer or calculator so that you become familiar with the hands-on methods. If you do not need hands-on knowledge of *MINITAB*, *MS Excel*, or the *TI-83/84 Plus*, you may choose to skip this section

and simply use the computer printouts or calculator screen captures for analysis as they appear in the text.

Most important, using statistics successfully requires common sense and logical thinking. For example, if we want to find the average height of all students at a particular university, would we select our entire sample from the members of the basketball team? In the body-temperature example, the logical thinker would question an 1868 average based on 1 million measurements—when computers had not yet been invented.

As you learn new statistical terms, concepts, and techniques, remember to view every problem with a critical eye and be sure that the rule of common sense applies. Throughout the text, we will remind you of the pitfalls and dangers in the use or misuse of statistics. Benjamin Disraeli once said that there are three kinds of lies: *lies*, *damn lies*, and *statistics*! Our purpose is to prove this claim to be wrong—to show you how to make statistics *work* for you and not *lie* for you!

As you continue through the book, refer back to this introduction every once in a while. Each chapter will increase your knowledge of statistics and should, in some way, help you achieve one of the steps described here. Each of these steps is important in achieving the overall objective of inferential statistics: to make inferences about a population using information contained in a sample drawn from that population.



# Describing Data with Graphs

# 1



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## How Is Your Blood Pressure?

Is your blood pressure normal, or is it too high or too low? The case study at the end of this chapter examines a large set of blood pressure data. You will use graphs to describe these data and compare your blood pressure with that of others of your same age and gender.

## LEARNING OBJECTIVES

Many sets of measurements are samples selected from larger populations. Other sets constitute the entire population, as in a national census. In this chapter, you will learn what a *variable* is, how to classify variables into several types, and how measurements or data are generated. You will then learn how to use graphs to describe data sets.

## CHAPTER INDEX

- Data distributions and their shapes (1.1, 1.3)
- Dotplots (1.3)
- Pie charts, bar charts, line charts (1.2, 1.3)
- Qualitative and quantitative variables—discrete and continuous (1.1)
- Relative frequency histograms (1.4)
- Stem and leaf plots (1.3)
- Univariate and bivariate data (1.1)
- Variables, experimental units, samples and populations, data (1.1)

### ● Need to Know...

How to Construct a Stem and Leaf Plot

How to Construct a Relative Frequency Histogram

## 1.1 Variables and Data

In Chapters 1 and 2, we will present some basic techniques in *descriptive statistics*—the branch of statistics concerned with describing sets of measurements, both *samples* and *populations*. Once you have collected a set of measurements, how can you display this set in a clear, understandable, and readable form? First, you must be able to define what is meant by measurements or “data” and to categorize the types of data that you are likely to encounter in real life. We begin by introducing some definitions.

### DEFINITION

A **variable** is a characteristic that changes or varies over time and/or for different individuals or objects under consideration.

For example, body temperature is a variable that changes over time within a single individual; it also varies from person to person. Religious affiliation, ethnic origin, income, height, age, and number of offspring are all variables—characteristics that vary depending on the individual chosen.

In the Introduction, we defined an *experimental unit* or an *element of the sample* as the object on which a measurement is taken. This is the same as saying that an *experimental unit* is the object on which a *variable* is measured. When a variable is actually measured on a set of experimental units, a set of measurements or **data** result.

### DEFINITION

An **experimental unit** is the individual or object on which a variable is measured. A single **measurement** or data value results when a variable is actually measured on an experimental unit.

If a measurement is obtained for every experimental unit in the entire collection, the resulting data set constitutes the *population* of interest. Any smaller subset of measurements is a *sample*.

### DEFINITION

A **population** is the set of all measurements of interest to the investigator.

### DEFINITION

A **sample** is a subset of measurements selected from the population of interest.

### EXAMPLE 1.1

A set of five students is selected from all undergraduates at a large university, and measurements are entered into a spreadsheet as shown in Figure 1.1. Identify the various elements involved in obtaining this set of measurements.

**Solution** The *experimental unit* on which the variables are measured is a particular undergraduate student on the campus, found in column A. Five *variables* are measured for each student: grade point average (GPA), gender, year in college, major, and current number of units

**Figure 1.1**  
Measurements on five  
undergraduate students

	A	B	C	D	E	F
1	Student	GPA	Gender	Year	Major	Number of Units
2	1	2	F	Fr	Psychology	16
3	2	2.3	F	So	Mathematics	15
4	3	2.9	M	So	English	17
5	4	2.7	M	Fr	English	15
6	5	2.6	F	Jr	Business	14

enrolled. Each of these characteristics varies from student to student. If we consider the GPAs of all students at this university to be the population of interest, the five GPAs in column B represent a *sample* from this population. If the GPA of each undergraduate student at the university had been measured, we would have the entire *population* of measurements for this variable.

The second variable measured on the students is gender, in column C. This variable is somewhat different from GPA, because it typically takes one of two values—male (M) or female (F). If we could identify each member of the population, it would consist of a set of Ms and Fs, one for each student at the university. The third and fourth variables, year and major, also involve nonnumerical data—year has four categories (Fr, So, Jr, Sr), and major has one category for each undergraduate major on campus. The last variable, current number of units enrolled, is numerically valued, consisting of a set of numbers rather than a set of qualities or characteristics.

Although we have discussed each variable individually, remember that we have measured each of these five variables on a single experimental unit: the student. Therefore, in this example, a “measurement” really consists of five observations, one for each of the five measured variables. For example, the measurement taken on student 2 produces this observation:

(2.3, F, So, Mathematics, 15)

There is a difference between a *single* variable measured on a single experimental unit and *multiple* variables measured on a single experimental unit as in Example 1.1.

#### DEFINITION

**Univariate data** results when a single variable is measured on a single experimental unit.

#### DEFINITION

**Bivariate data** results when two variables are measured on a single experimental unit.  
**Multivariate data** results when more than two variables are measured.

If you measure the body temperatures of 148 people, the resulting data are *univariate*. In Example 1.1, five variables were measured on each student, resulting in *multivariate* data.

## Types of Variables

Variables can be classified into one of two types: **qualitative** or **quantitative**.

#### DEFINITION

**Qualitative variables** measure a quality or characteristic on each experimental unit.  
**Quantitative variables** measure a numerical quantity or amount on each experimental unit.